Diagrammatic sets and rewriting in weak higher categories

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There is a draft, but I am rewriting it from scratch.
Some definitions have changed.
Some results I will mention do not hold with the old definitions.
The new version should be out before the end of the month.

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$\rightsquigarrow$ Segal spaces, complicial sets... pick your favourite.

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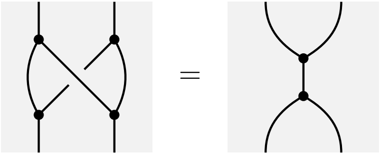
Bialgebra equation



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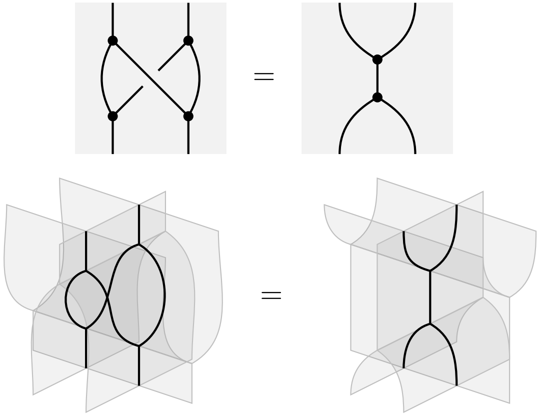

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An interaction of *planar* (2d) diagrams,

producing a transformation of 3d diagrams

(a 4d diagram)

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How do we interpret this?

Pasting theorems

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The golden age of strict ω-categories

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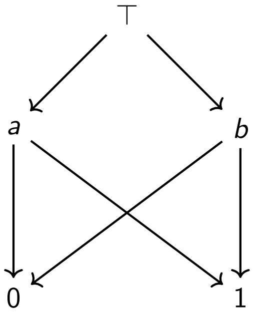
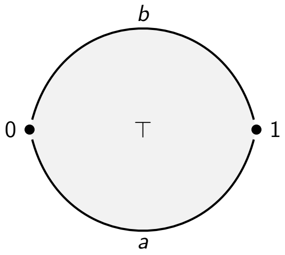
Then several works on the combinatorics of *pasting diagrams* and their *pasting theorems* in strict n-categories:

- **1988**: John Power
- **1989**: Michael Johnson
- **1991**: Ross Street, John Power
- **1993**: Richard Steiner

Directed complexes

We can associate to a cell complex its face poset...





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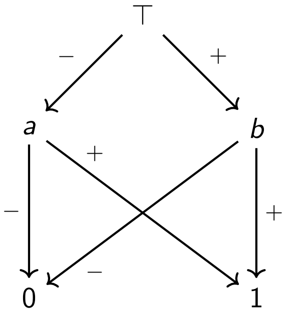

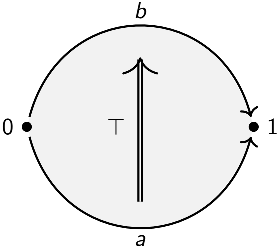

and to a pasting diagram its oriented face poset.

Technical interlude #1: Directed complexes

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- An *orientation* on a finite poset P is an edge-labelling $o : \mathcal{H}P_1 \to \{+, -\}$ of its Hasse diagram.
- An *oriented graded poset* is a finite graded poset with an orientation.
- If $U \subseteq P$ is (downward) closed, $\alpha \in \{+, -\}$, $n \in \mathbb{N}$,

$$\Delta_n^\alpha U := \{x \in U \,|\, \dim(x) = n \text{ and if } y \in U \text{ covers } x \text{, then } o(y \to x) = \alpha\},$$
$$\partial_n^\alpha U := \mathrm{cl}(\Delta_n^\alpha U) \cup \{x \in U \,|\, \text{for all } y \in U \text{, if } x \leq y \text{, then } \dim(y) \leq n\},$$
$$\Delta_n U := \Delta_n^+ U \cup \Delta_n^- U, \qquad \partial_n U := \partial_n^+ U \cup \partial_n^- U.$$

Technical interlude #1: Directed complexes

If U is a closed subset of P, then U is a *molecule* if either

- U has a greatest element, in which case we call it an *atom*, or
- there exist molecules U_1 and U_2, both properly contained in U, and $n \in \mathbb{N}$ such that $U_1 \cap U_2 = \partial_n^+ U_1 = \partial_n^- U_2$ and $U = U_1 \cup U_2$.

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An oriented graded poset P is a *directed complex* if, for all $x \in P$ and $\alpha, \beta \in \{+, -\}$, if $n = \mathrm{dim}(x)$,

1. $\partial^\alpha x$ is a molecule, and
2. $\partial^\alpha(\partial^\beta x) = \partial_{n-2}^\alpha x$.

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All directed complexes present ω-categories —
fewer present polygraphs,
that is,
ω-categories that are freely generated by some of their cells.

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A variant of this was used to define
the Gray product of ω-categories
(Steiner 2004, Ara-Maltsiniotis 2017)

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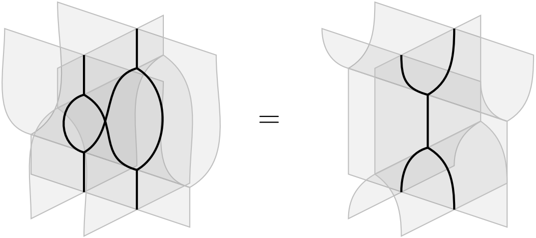
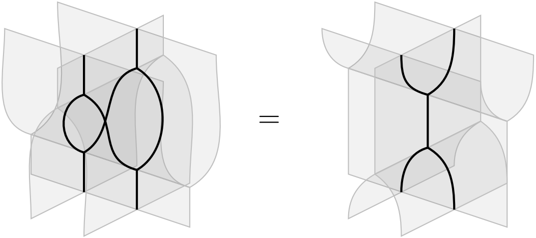
Gray products and diagrammatic algebra



2d + 2d = 4d

Around this time, I start seeing Gray products everywhere in diagrammatic algebra

Gray products and diagrammatic algebra



$$2\text{d} + 2\text{d} = 4\text{d}$$

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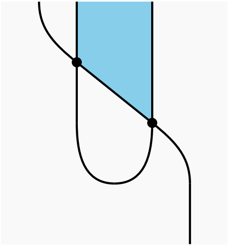
(Fortunately I was not the only one)

Gray products and diagrammatic algebra

Example: Biunitary equations

Used by Jamie Vicary and Mike Stay to unify quantum and encrypted communication protocols. They are models of a Gray product of 2-categories.



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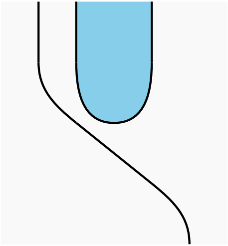
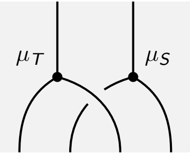


Gray products and diagrammatic algebra

Example: Distributive laws of monads

They are models in **Cat** of a Gray product of 2-categories.



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PRO $\wedge$ **PRO** $\rightsquigarrow$ 4-cat with one 0-cell, one 2-generator

Morally this should be a *braided monoidal category*.
But in strict ω-categories, it is a *commutative monoidal category*.
This breaks everything.

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The core of the argument relies on the fact that "doubly monoidal" degenerates to "commutative" in strict 3-categories (strict Eckmann-Hilton).

...still contained some good ideas

Good takeaway #1 from Kapranov-Voevodsky:

homotopy types may have **semi***strict algebraic models with weak units*

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- **2018**: Henry proves the homotopy hypothesis for "regular ω-groupoids".

Diagrams with spherical boundary

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$\sim$ "are homeomorphic to n-balls"

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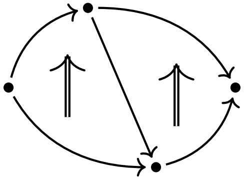
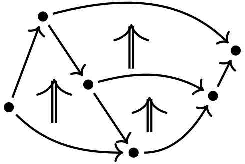
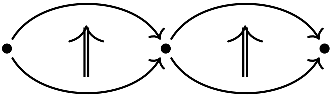




but not



Technical interlude #2: Spherical boundary

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*simplicial nerve of poset + realisation of simplicial sets

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More in general, let $\mathcal{C}$ be a class of molecules closed under isomorphism, boundaries, and inclusion of atoms, and included in the class $\mathcal{S}$ of (regular) molecules with spherical boundary.

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More in general, let $\mathcal{C}$ be a class of molecules closed under isomorphism, boundaries, and inclusion of atoms, and included in the class $\mathcal{S}$ of (regular) molecules with spherical boundary.

- A *$\mathcal{C}$-directed complex* is a directed complex whose atoms are all in $\mathcal{C}$.

...and more good ideas

Good takeaway #2 from Kapranov-Voevodsky:

...and more good ideas

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Diagrammatic sets

...and more good ideas

Good takeaway #2 from Kapranov-Voevodsky:

Diagrammatic sets

Kapranov-Voevodsky pass from spaces to ω-categories
through an intermediate notion
of "spaces locally modelled on combinatorial pasting diagrams",

they call diagrammatic sets.

Diagrammatic sets

- **2019**: Kapranov-Voevodsky's equivalence of “Kan diagrammatic sets” and spaces is “morally correct”

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Regular molecules with spherical boundary works.

But we take a more axiomatic approach.

Technical interlude #3a: Morphisms of directed complexes

A *map* $f : P \to Q$ of $\mathcal{C}$-directed complexes is a function that satisfies

$$\partial_n^\alpha f(x) = f(\partial_n^\alpha x)$$

for all $x \in P$, $n \in \mathbb{N}$, and $\alpha \in \{+, -\}$.

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A map factors essentially uniquely as a *surjection* followed by an *inclusion*.

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Let $f : P \to Q$ be a map. Then f is a closed, order-preserving, dimension-non-increasing function of the underlying posets.

Technical interlude #3a: Morphisms of directed complexes

A $\mathcal{C}$*-functor* $f : P \nrightarrow Q$ of $\mathcal{C}$-directed complexes is a function $f : \mathcal{C}\ell(P) \to \mathcal{C}\ell(Q)$ such that

1. f preserves all unions and binary intersections,
2. $\partial_n^\alpha f(\mathrm{cl}\{x\}) = f(\partial_n^\alpha x)$, and
3. $f(\mathrm{cl}\{x\})$ is a $\mathcal{C}$-molecule

for all $x \in P$, $n \in \mathbb{N}$, and $\alpha \in \{+, -\}$.

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1. f preserves all unions and binary intersections,
2. $\partial_n^\alpha f(\mathrm{cl}\{x\}) = f(\partial_n^\alpha x)$, and
3. $f(\mathrm{cl}\{x\})$ is a $\mathcal{C}$-molecule

for all $x \in P$, $n \in \mathbb{N}$, and $\alpha \in \{+, -\}$.

A class $\mathcal{C}$ is *algebraic* if $\mathcal{C}$-functors compose. We assume that $\mathcal{C}$ is algebraic.

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A $\mathcal{C}$-functor factors e.u. as a *subdivision* followed by an *inclusion*.

Technical interlude #3a: Morphisms of directed complexes

A span of inclusions of subcategories:

$$\mathbf{DCpx}^{\mathcal{C}} \hookleftarrow \mathbf{DCpx}^{\mathcal{C}}_{in} \hookrightarrow \mathbf{DCpx}^{\mathcal{C}}_{fun}$$

Technical interlude #3b: Convenient classes

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The class $\mathcal{S}$ is convenient!

Diagrammatic sets

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- It is *composable* if $U \in \mathcal{C}$, and a *cell* if U is an atom.

Fixing half of KV's proof

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There is a realisation of Kan diagrammatic sets that is surjective on homotopy types,
together with natural isomorphisms between the homotopy groups of a pointed Kan diagrammatic set and those of its realisation.

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which brought me to Paris.

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polygraphs and *CW complexes*,
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The silver age of strict ω-categories

Many of the core ideas in polygraphic rewriting
rest on an analogy between

polygraphs and *CW complexes*,
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This analogy is limited by
the fact that strict ω-categories do not model all spaces.

A suggestion: rewriting in diagrammatic sets

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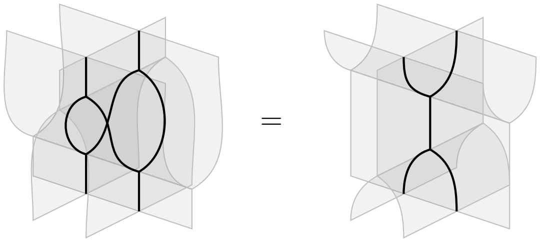
5. Gray products and joins are easily defined and computed

A suggestion: rewriting in diagrammatic sets



The smash product of pointed diagrammatic sets produces this equation, the way it should.

Equivalences and weak composites

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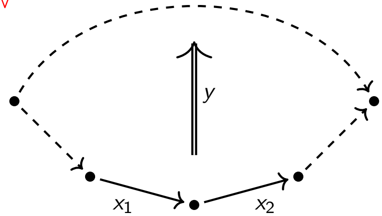
“Stuff” a diagram with units and it becomes regular.

Equivalences and weak composites

If $(x_1, x_2) \Rightarrow \lfloor x_1, x_2 \rfloor$ exhibits $\lfloor x_1, x_2 \rfloor$ as a weak composite:

$\forall$



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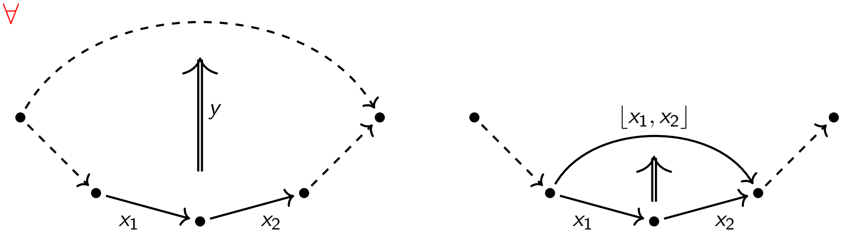

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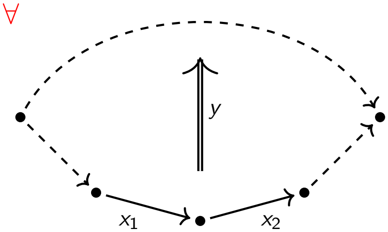

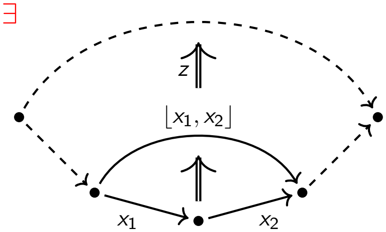

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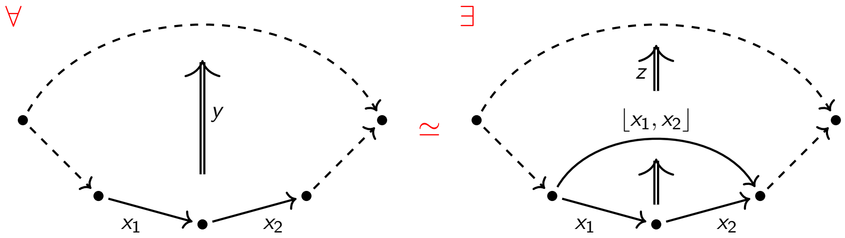

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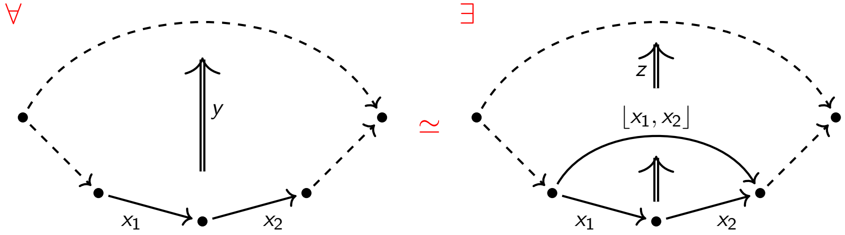

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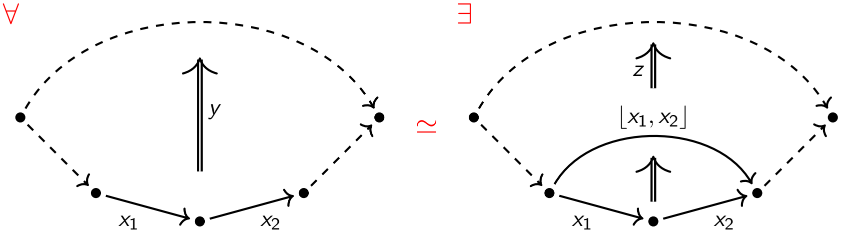

And this equivalence should be witnessed by **3-dimensional equivalence diagrams**...

whose definition involves 4-dimensional equivalence diagrams, etc

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Properties of equivalences:

- All *degenerate* composable diagrams are equivalences.

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- In a Kan diagrammatic set, all composable diagrams are equivalences.

A semistrict algebraic model

In the span

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the two functors preserve the set Γ of colimit diagrams containing the initial object and all pushouts of inclusions.

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the two functors preserve the set Γ of colimit diagrams containing the initial object and all pushouts of inclusions.

$\odot\mathbf{Set}$ is equivalent to the category $\mathrm{PSh}_{\Gamma}(\mathbf{DCpx}^{\mathcal{C}}_{fun})$ of Γ-continuous presheaves on $\mathbf{DCpx}^{\mathcal{C}}$.

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Applying $\mathrm{PSh}_{\Gamma}(-)$, we obtain a cospan

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of restriction functors, where $\mathbf{Pol}^{\mathcal{C}} := \mathrm{PSh}_{\Gamma}(\mathbf{DCpx}^{\mathcal{C}}_{in})$ and $\omega\mathbf{Cat}^{\mathcal{C}}_{nu} := \mathrm{PSh}_{\Gamma}(\mathbf{DCpx}^{\mathcal{C}}_{fun})$.

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- $\mathbf{Pol}^{\mathcal{C}}$ is a category of "combinatorial $\mathcal{C}$-polygraphs" (only faces, no units or compositions)
- $\omega\mathbf{Cat}^{\mathcal{C}}_{nu}$ is a category of "non-unital $\mathcal{C}$-ω-categories" (only faces and compositions, no units)

A semistrict algebraic model

Units and compositions interact nicely separately with faces. If they are let to interact fully with each other, they produce strict Eckmann-Hilton.

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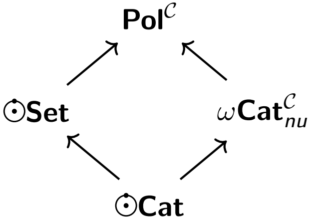
- Idea: put them together with only a modicum of interaction.



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Idea: take a unit on a composable diagram, and fully compose the boundary only on one side.

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Say that $\mathcal{C}$ is *algebraically free* if all $\mathcal{C}$-directed complexes present polygraphs.

*If $\mathcal{C}$ is algebraically free, then ω***Cat** *embeds as a full subcategory into $\odot$***Cat**.

Two conjectures

1. Conjecture: *If X is a diagrammatic set with weak composites, its inclusion in the free diagrammatic ω-category on X is a weak equivalence.*

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2. Conjecture: *Every convenient class $\mathcal{C}$ is algebraically free.*

Directed homotopy theory: a tinkerer's approach

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The appearance of smash products in diagrammatic algebra seems to me another piece of a puzzle.

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Directed homotopy theory: a tinkerer's approach

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Work in progress:
a model of computation in diagrammatic sets
based on a "directed homotopy extension property".

⋮

2014

2015

2016

2017

2018

2019

2020

Thanks for listening!